Kinematics

In this unit you are going to develop the following skills:

- to understand the differences between scalar and vector quantities
- to know the main physical quantities related to motion: position, displacement, velocity and acceleration
- to understand how acceleration can change both speed or direction of the motion
- to plot different graphs of motion and get information from them
- to describe circular uniform motion by means of its period or frequency
- to solve quantitative exercises related to linear or circular motions

1. Introduction to Kinematics

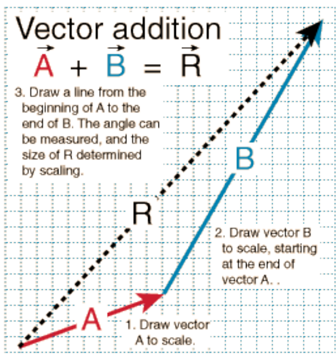
Physics is a science, which studies the whole Universe: the motion of particles, light, sound, energy, electricity, etc. ***Kinematics*** is a branch of Physics, which describes the motion of bodies, without regard to its causes

Physicists deal with ***physical quantities***, any property which can be measured. Velocity, forces, electric current, temperature are different examples of physical quantities.

Physical quantities can be classified into two types: ***scalars*** and ***vectors***. A physical quantity is scalar when it is fully described by just its magnitude. Temperature, energy or pressure are examples of scalar quantities.

On the other hand, vectors are quantities, which have magnitude and direction. In other words, a vector quantity depends on its direction. Forces are vectors because we can pull an object upwards or rightwards. Vector quantities are usually represented with an arrow over its symbol: $\vec{F}$.

Vectors can be added or subtracted, but mathematical rules are different. We can add two vectors graphically drawing the second vector at the end of the first one. The sum of both vectors or the resultant vector is a vector which has the beginning of the first vector and the end of the second one.



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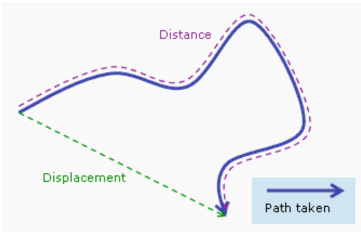
- ***Activity 1***: *We drive far away from Madrid at 120 km/h. Do you know our position after one hour driving?. What kind of information do you need to determine perfectly our location?*
- ***Activity 2***: *Classify the following quantities into scalars or vectors: mass, velocity, force, length, electrical charge and energy*

2. Position and displacement

Scientists try to answer the questions we raise about the Universe. One of the first questions we have to solve is "where a body is". First of all we have to simplify the dimensions of the body so we have to consider it as a ***particle***, which means a point where all the mass of the object is placed. Then we have to choose a special point, from which every distance has to be measured. This point is called the ***origin*** of our ***frame of reference***.

Now we can define the ***position*** of the particle, which is the distance from the origin of our frame of reference to the point where our particle is placed. Position is a vector quantity, which refers to the location of a particle and it is measured in ***metres***. On a general basis, it is represented by the symbol ***s*** –when the motion is 1-dimensional- or $\vec{r}$ **.**



Motion can be defined as any change of location or position of a particle. Consider a particle, which is moving from a point A to a point B. If we liked to know how long the motion is, we would define a new quantity to measure the length travelled by this particle. We can distinguish two different quantities: distance and displacement. On the one hand, ***distance*** is a scalar quantity defined as the total length covered by a particle through its path. On the other hand, ***displacement*** is vector quantity, which is the shortest distance between A and B. Displacement refers to the distance covered by a particle through a direction and it´s measured in metres. It is represented by the symbol Δs or $\Delta\vec{r}$

Despite of its vector characteristics, displacement gives a valuable information about the motion of a particle: a non-zero displacement means that a particle has moved from a point to a different one. On the other hand, zero-displacement means an object at rest. Besides, the value of its displacement refers to the distance travelled by the particle when the time interval is short.

- $\Delta\vec{r} \neq 0$ ***Motion***
- $\Delta\vec{r} = 0$ ***Rest***

3. Speed and velocity

Sometimes we don´t measure distances in metres, but in units of time, because following the fastest way could be better than the shortest one. As a motion can be considered fast if it covers great distances in short intervals of time, we need to define a new quantity which is directly proportional to distance covered and inversely proportional to time taken. In other words it must be a ratio of distance travelled to time taken.

As we have done with distance and displacement, we´ll distinguish between two different quantities: speed and velocity. ***Average speed*** is a scalar quantity which is the relationship between distance and time. It is measured in metres per second: $m.s^{-1}$. Speed is the rate of change of distance and refers to how fast a particle is moving: a fast-moving particle has high speed and covers large distances in a short interval of time. On the other hand, zero-speed means a ***state of rest***.

$$v_{average} = \frac{\Delta s}{\Delta t}$$

Generally speaking average speed is not the same speed at which an object is moving. When we define the average speed through a short interval of time we get the actual speed of the object, in other words, the ***instantaneous speed***.

Velocity is a vector quantity, which means that is direction aware. ***Average velocity*** is the rate of change of position or, in other words, the ratio of displacement to time. It is also measured in metres per second: $m.s^{-1}$.

$$\vec{v}_{average} = \frac{\Delta \vec{r}}{\Delta t}$$

Speed and velocity are related quantities, and sometimes they are almost synonyms. However, velocity is a vector quantity, it has both magnitude and direction, so it´s the most suitable quantity in Physics.

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- ***Activity 3***: *We drive far away from Madrid. First of all we start moving at 100 km/h for 30 minutes, but we arrive to a traffic jam and we stop for 40 minutes. Finally we covered the last 30 km at 90 km/h. Calculate the average speed of the car*

We can use velocity instead of displacement to describe the motion of a particle: zero-velocity means that the particle is at rest but non zero-velocity means a moving object.

- $v \neq 0$ ***Motion***
- $v = 0$ ***Rest***

4.- Acceleration

When a particle is moving, it usually undergoes changes of velocity: first, it can speed up, then it can change of direction and finally, it can slow down. It is very important to measure the changes of velocity so as to know the type of motion.

Acceleration is a vector quantity which refers to the rate of change of velocity. It is defined as the relationship between the change of velocity and the time taken. Notice that is directly proportional to the change of velocity and inversely proportional to the time taken. Its is measured in metres per second squared

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t}$$

- ***Activity 4***: *We drive far away from Madrid at 120 $km.h^{-1}$. You throw a coin vertically upwards and it goes upwards and falls down after a second. As our car is moving horizontally at the same time, it covers 30 metres per second. Do you expect that the coin will fall down backwards?. Can you imagine a frame in which the coin falls down to the same initial point?*

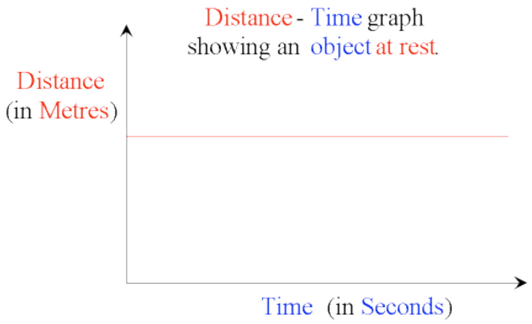
As velocity is a vector, which describes the speed and its direction, an accelerated particle can change both factors: a ***change of speed*** (speeding up or slowing down) or a ***change of direction***.

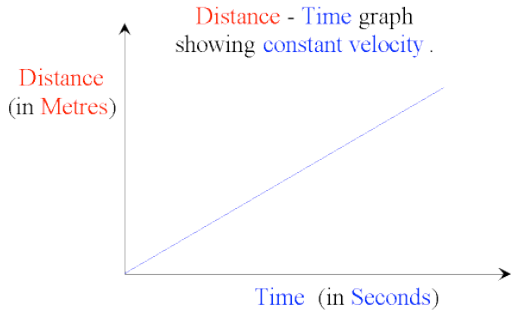
5.- Uniform linear motion

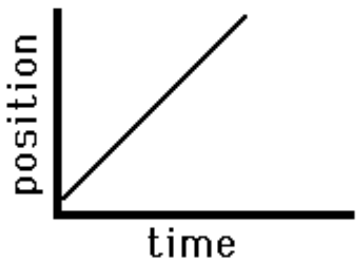
Uniform linear motion (ULM) is a motion along a straight line which has constant velocity. As its velocity is constant, it has zero-acceleration. A body wich is moving at constant velocity travels the same distance each time interval.

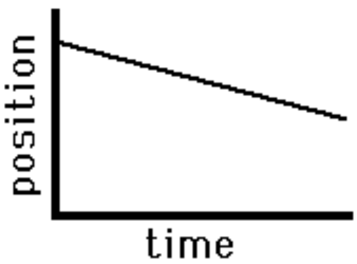
$$v = cte \qquad a = 0$$
$$\mathbf{s = s_o + v.t}$$

We can represent position or velocity versus time and get a lot of information from this kind of diagrams. In a ULM the position-time diagram is a straight line. The slope (or gradient) of the line is the velocity of the motion, so the greater the slope, the faster the motion is.









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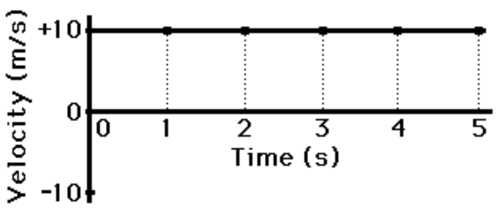
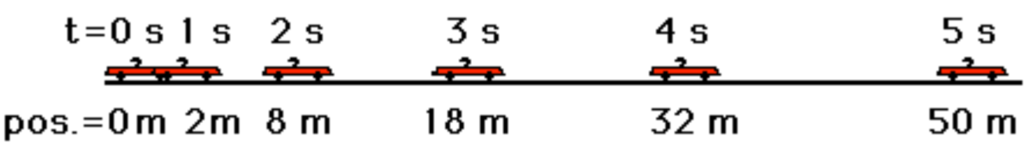


Notice that positive or negative velocities refers to leftwards or rightwards motion. Velocity-time graphs are horizontal lines. Besides total distance travelled by a particle is the area between the graph and the baseline in velocity – time diagrams.

5.- Linear uniformly accelerated motion

Uniformly accelerated motion is a motion along a straight line which has constant acceleration



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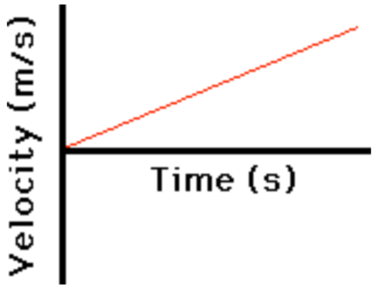
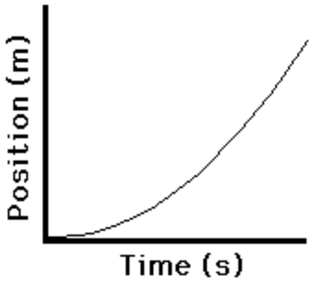
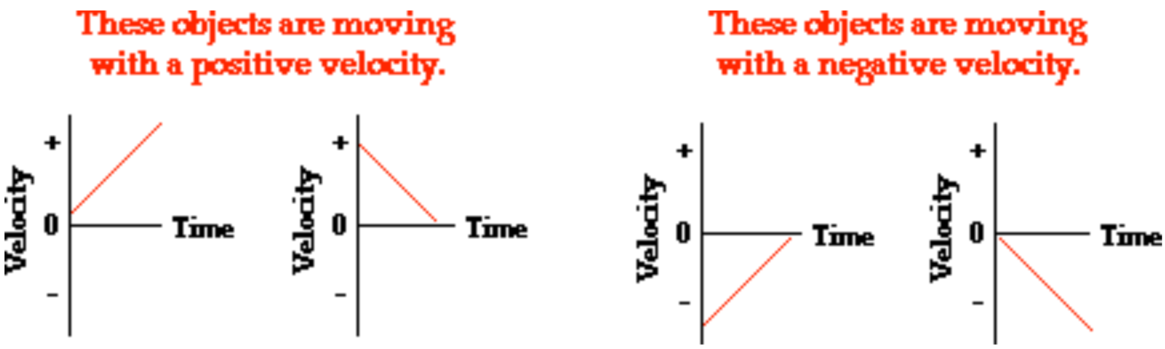
Here you have the Kinematics equations for LUAM:

$$a = cte$$
$$\mathbf{s = s_o + v_o \cdot t + \tfrac{1}{2} \cdot a \cdot t^2}$$
$$\mathbf{v = v_o + a \cdot t}$$
$$\mathbf{v^2 = v_o^2 + 2 \cdot a \cdot s}$$

Velocity-time graphs are different: a straight line whose slope refers to acceleration. On the other hand, position-time graphs are parabolas in which slope increases or decreases as time passes. Notice that the slope in a parabola increases because the particle is speeding up.







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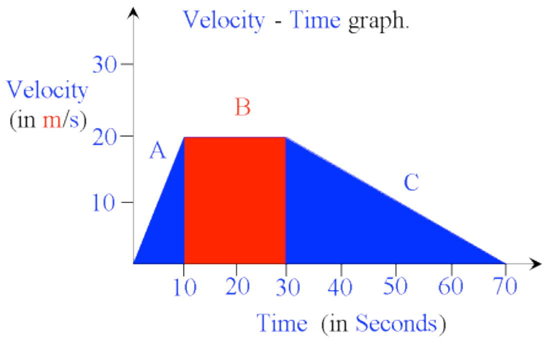
Positive slopes refers to positive acceleration, or, in other words, a particle which is speeding up. On the other hand, negative gradient represent a slowing down motion.

As we have explained before, the area between the graph and the baseline is the total distance travelled by the particle.

In this diagram we find three different stages through the motion: first he particle speeds up; then it goes on moving at the same rate; and finally it slows down until it stops. Total distance is the coloured area in the graph.



5.- Uniform circular motion

Uniform circular motion is a motion through a circumference at the same rate. In other words, the particle covers the same distance each interval of time. Notice that the speed of the motion is always the same, but velocity changes because of the change of direction along the circumference.

As the speed of the particle doesn´t change, the body always needs the same interval of time to cover the whole circumference. In other words, uniform circular motion is a ***periodic***

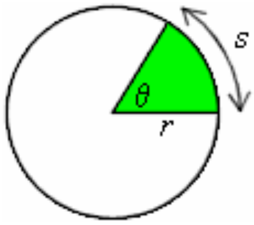
motion, which means that it is regular: the particle arrives to the same position after this interval of time, ***Period*** is the time for the particle to cover a whole circumference. It is represented as T and it´s measured in seconds.

Frequency is the number of cycles finished in a unit of time. So we can notice that frequency is the reciprocal of period. The unit for frequency is cycles.s^{-1} or Hertz.

$$T = \frac{1}{\nu}$$

Circular motion must be described using new quantities: every point of a circumference has the same distance to the centre, which is the radius, so we prefer to determine the position of the body using an angle. Angle is a quantity which refers to the space between two straight lines which have one common point

Angle is measured in radians. One ***radian*** is the angle which has the length of its arc equal to its radius. Therefore, we can calculate the angle measured in radians dividing the length of the arc, s, by its radius, r. You have to notice that radian is adimensional because is a proportion, a conversion factor between two lengths, the arc and the radius



$$\theta = \frac{s}{R} \Rightarrow s = \theta . R$$

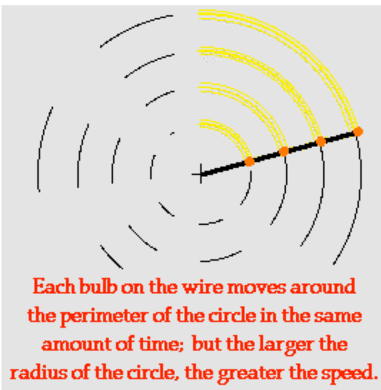
- ***Activity 5***: *Do you know the Global Positioning System on Earth? Which is the quantity that you need to know to place your position on Earth?*

Remember that the perimeter of a circle is 2π.r. The angle defined by the whole circumference can be calculated dividing its perimeter by its radius, so it is 2π

2π rad = 360º π rad = 180º

As we have already defined speed and velocity for linear motions, we can define a new velocity, ***angular velocity***, which refers to the rate of a circular motion. Angular velocity is the angle swept by a particle which is turning around a circle divided by the time taken.

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \theta = \omega . t$$



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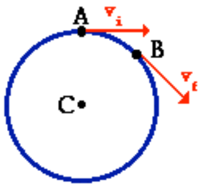
It is measured in radians per second, rad.s^{-1}. Consider a particle which orbits a whole circumference in a period T, we can determine its angular velocity as

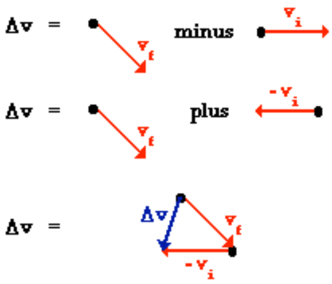
$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

As velocity and angular velocity are both quantities, which refer to rate of the motion, there must be a relationship between both quantities.

$$s = \theta . R \Rightarrow \frac{s}{t} = \frac{\theta}{t} . R \qquad v = \omega . R$$

Notice that linear speed depends on angular velocity and radius as well: the greater the radius, the faster the motion





Despite of moving at the same speed, a particle with a circular uniform motion has acceleration, because the direction of velocity changes as time passes. This acceleration is called ***centripetal acceleration*** and acts towards the centre of the circle. It depends on the speed of the motion and the radius of the circumference

$$a_n = \frac{v^2}{r} = \omega^2.r$$